

# Conservation of Angular Momentum During Human Locomotion

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**The Problem:** Our goal is to describe stable human locomotion, including walking and running, using the concept of conservation of angular momentum about the center of mass (CM) of the body.

**Motivation:** The mechanics of walking and running are extremely complicated, exemplified by the fact that no robots can perform robust human walking. By focusing on angular momentum, a fundamental physical concept, we hope to develop a relatively simple model of stable motion. The angular momentum of a system is conserved if no external torques act on the system. During the aerial phase of locomotion, angular momentum is obviously conserved, but the interaction of the feet with the ground introduces external torques on the body. Therefore, there is no *a priori* reason for angular momentum in the CM frame to be conserved. However, recent observations in our research indicate that angular momentum is conserved to a large extent.

**Previous Work:** Surprisingly, very little research has been done with regard to angular momentum during human locomotion. In 1986 Raibert mentioned the idea that “a control system that keeps the angular momentum constant during stance could achieve higher efficiency and better performance [2],” but this approach has not been explored thoroughly.

**Approach:** Our approach was to analyze real human locomotion data gathered in the Gait Laboratory of Spaulding Rehabilitation Hospital. The Gait Laboratory has facilities to obtain position data from markers placed at various parts of the body as well as ground reaction force and center of pressure (CP) measurements. One consequence of the conservation of angular momentum in the CM frame is that the radial component  $F_r$  of the ground reaction force is proportional to the distance  $r$  between the CP and the CM projection onto the ground:  $F_r = \frac{-F_z}{z} r$  where  $z$  is the vertical distance between the CM and the ground and  $F_z$  is the vertical component of the ground reaction force. We obtained excellent agreement between the force predicted by this model and the force measured in the experiment (Fig. 1).

In order to assign a quantitative value to the degree of angular momentum conservation, we found the adjusted coefficient of multiple determination  $R_a^2$ , which lies between zero and one and evaluates the similarity of waveforms across many trials, with one being 100% similar and zero being 0% similar [1]. We found  $R_a^2$  to be 0.96 for standing, 0.83 for walking, and 0.69 for running. A larger  $R_a^2$  value means a higher degree of angular momentum conservation.

**Impact:** Besides providing an elegant model for human locomotion, the conservation of angular momentum makes possible a robust control algorithm for robotic walking. The fact that the human body highly conserves angular momentum during locomotion suggests that this is a high-level control objective. The conservation of angular momentum as a global control task could produce stable robotic locomotion.

**Future Work:** Attempts to calculate the angular momentum directly instead of looking at only torques were unsatisfactory because we did not have a sufficiently accurate model of the human body. Our next goal is to develop such a model using Creature Library and SD/FAST, feed trajectory data obtained from position marker data into the model, and use the simulation to find angular momentum and to corroborate our earlier results. We will also explore how angular momentum is partitioned throughout the body, and this partition will then be the basis of a control system that explicitly controls the angular momentum of individual body segments.

**Research Support:** Funding was provided by the Schaeffer Foundation and the Paul E. Gray UROP Fund.

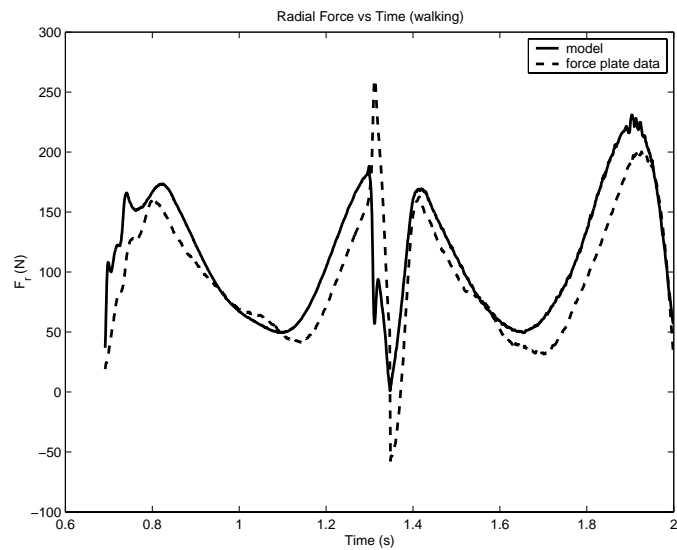


Figure 1: Model and experimental forces during walking.

**References:**

- [1] M. P. Kadaba et. al. Repeatability of kinematic, kinetic, and electromyographic data in normal adult gait. *Journal of Orthopedic Research*, pages 849–860, 1989.
- [2] Marc H. Raibert. *Legged Robots that Balance*. MIT Press, 1986.